

Problem 1

Find

$$\int \frac{5x^2 + 20x + 4}{x^3 + 2x^2 + x} dx$$

Answer:

$$\ln \left| \frac{x^6}{x+1} \right| - \frac{9}{x+1} + C$$

Problem 2

Solve

$$\int x^3(4+x^2)^{\frac{3}{2}} dx$$

using the substitution $x = 2 \tan \theta$.

Answer:

$$128 \left(\frac{(4+x^2)^{\frac{7}{2}}}{2^7 7} - \frac{(4+x^2)^{\frac{5}{2}}}{2^5 5} \right) + C$$

problem 3

Write the volume under the surface $z = 1 - x^2 - y^2$ and above the triangle: $x \geq 0$, $y \geq 0$, $x + y \leq 1$ as a double integral and compute it.

Answer:

$$\frac{2}{3}$$

Problem 4

Compute the volume under the surface $z = x^2 \sin y^4$ and above the triangle in the xy plane with vertices $\{(0, 0), (0, \pi^{\frac{1}{4}}), (\pi^{\frac{1}{4}}, \pi^{\frac{1}{4}})\}$.

Answer:

$$= \frac{-1}{6\pi^{\frac{3}{4}}}$$

problem 5

Evaluate

$$\int \frac{1}{\sqrt{2x-x^2}} dx$$

Answer:

$$\sin^{-1}(x-1) + C$$

Problem 6

Compute

$$\int (x^2 + 3x) \sin x dx.$$

Answer:

$$\int (x^2 + 3x) \sin x dx = -(x^2 + 3x) \cos x + (2x + 3) \sin x - 2 \cos x$$

Problem 7

Show whether the following series converges or diverges, if it converges show whether it converges absolutely or conditionally.

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{(n+1)\ln(n+1)}$$

Answer:

The series $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{(n+1)\ln(n+1)}$ converges conditionally.

Problem 8

Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^3}$$

Answer: Thus the series converges.

Problem 8

Compute the arclength, L , of the following curve:

$$c(t) = (e^t \cos t, e^t \sin t), \quad 0 \leq t \leq 2\pi.$$

Answer:

$$L = \sqrt{2}(e^{2\pi} - 1)$$

Problem 9

Find the interval of convergence of the following power series:

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{\sqrt{n+1}}$$

Answer:

$$x \in (-1, 1)$$

Problem 10

Find the interval of convergence for the following series:

$$\sum_{n=1}^{\infty} \frac{e^n(4-x)^n}{n^3}$$

Answer:

$$x \in [4 - e^{-1}, 4 + e^{-1}]$$

Problem 11 For what values of k does the following integral converge?

$$\iint_D \frac{dA}{(x^2 + y^2)^k}, \quad D = \{(x, y) : x^2 + y^2 \leq 1\}$$

Answer:

$$k < 1$$

Problem 12

Evaluate the following triple integral:

$$I = \int \int \int_{\mathbb{R}^3} e^{-x^2-2y^2-3z^2} dV$$

Answer:

$$\sqrt{\frac{\pi^3}{6}}$$

Problem 13

For which values of x does the following series converge?

$$\sum_{n=1}^{\infty} \frac{1}{n^{x^2-2x-1}}$$

Answer:

$$x > 1 + \sqrt{3}$$

Problem 14

Evaluate the following series:

$$S = \sum_{n=0}^{\infty} \frac{3 + 2^n}{3^{n+2}}$$

Answer:

$$\frac{5}{6}$$

Problem 15

Evaluate the following series:

$$S = \sum_{n=1}^{\infty} 3 \left(\frac{-1}{4} \right)^{n-1}$$

Answer:

$$\frac{12}{5}$$

Problem 16

Evaluate the following series:

$$S = \sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$$

Answer:

$$\frac{1}{2}$$

Problem 17

Compute

$$I = \int \int \int_D (3 + 2xy) dV, \text{ where } D \text{ lies above } z \geq 0 \text{ and below } x^2 + y^2 + z^2 = 4$$

Answer:

$$I = 16\pi$$

Problem 18

Evaluate

$$\int \int \int_R (x^2 + y^2 + 2z^2) dV \text{ where } R \text{ is the cylinder } 0 \leq x^2 + y^2 \leq a^2, 0 \leq z \leq h$$

Answer:

$$\frac{\pi a^4 h}{2} + \frac{2\pi a^2 h^3}{3}$$