Math 211 Review Session

1. Solve the following system of equations:

$$\begin{array}{rrrrrcr} 3x_1 & +6x_2 & +x_3 & -6x_4 & -14x_5 & = & 24 \\ 3x_1 & +6x_2 & +2x_3 & -4x_4 & -9x_5 & = & 18 \\ -2x_1 & -4x_2 & -x_3 & +4x_4 & +9x_5 & = & -16 \end{array}$$

2. Let $A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ -1 & 0 & 2 \end{bmatrix}$.

   (a) Find A^{-1}.

   (b) Solve the system $AX = B$, where $B = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

3. Suppose that A, B, and C are 3×3 matrices such that $\det A = 2$, $\det B = 3$ and $\det C = -2$. Find the value of each of the following:

   (a) $\det(3A^T B^{-1} C)$

   (b) $\det(B \operatorname{adj} B)$

4. Let $B = \begin{bmatrix} -1 & 2 & 3 \\ 0 & -5 & -6 \\ 0 & 2 & 2 \end{bmatrix}$. Find the characteristic polynomial, eigenvalues and eigenvectors of B. If possible, find a diagonal matrix D and an invertible matrix P such that $P^{-1}AP = D$.

5. Write the standard matrix for the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that consists of a counterclockwise rotation by $\frac{3\pi}{2}$ followed by a reflection in the line $y = 2x$.

6. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$. Find the standard matrix for T.

7. Solve for z. Write your answer in the form $z = a + bi$:

$$(3 + 4i)z - (\overline{1 - 3i}) = -2i$$

8. Find all complex numbers z such that $z^3 = 8i$

9. Find an equation for the plane passing through the points $A(1, -1, 1)$, $B(3, 0, 1)$ and $C(-1, 2, 5)$.

10. Let P be the point $(1, 8, -3)$ and let L be the line described by the equations $x = 1+t,\ y = 3-2t,\ z = -1+t$.

    (a) Find the shortest distance between P and L.

    (b) Find the point on L which is closest to P.